Let $M$ be a Smooth manifold of arbitrary dimension $n \geq 3$ and $\Omega$ be a skew-symmetric 2-form on manifold $M$. A Radical of 2-form $\Omega$ at the point $x \in M$ is a subspace $\text{rad} \Omega_x$ of tangent space $T_x M$ formed as

$$\text{rad} \Omega_x = \{v \in T_x M : \Omega_x(v, .) = 0\},$$

where $\Omega(v, .)$ denote that second vector argument can be an arbitrary vector from $T_x M$. Note that an radical can be also well-defined for any bilinear form at the point $x$. It is easy to view that at the point $x$ always exists a tangent subspace $D_x$ so that restriction of 2-form $\Omega_x$ to $D_x$ be a non-degenerated 2-form, and $\text{dim}(D_x)$ always be a even number for any $n$. This tangent subspace is called the Work Subspace.

An Affinor associated with 2-form $\Omega_x$ at the point $x \in M$ is an endomorphism $\Phi_x$ of tangent space $T_x M$ that satisfies the following conditions:

1) $\ker \Phi_x = \text{rad} \Omega_x$;

2) $\Phi_x^2 |_{D_x} = -\text{id}$, where $\text{id}$ is an identity operator in $D_x$;

3) $\Omega_x \circ \Phi_x = \Omega_x$;

4) For all $v \in T_x M \quad \Omega_x(v, \Phi_x w) \geq 0$.

The properties of affinor $\Phi_x$ implies that bilinear form

$$\Omega_x(v, \Phi_x w), v, w \in T_x M$$

is the Inner Product in Work Subspace $D_x$, and equals to zero when whether $v$ or $w$ bellong to $\text{rad} \Omega_x$.

A Radical Metric for 2-form $\Omega_x$ at the point $x \in M$ is a symmetric 2-form $\beta_x$ so that $\text{rad} \beta_x = D_x$ and restriction of $\beta_x$ to $\text{rad} \Omega_x$ is an Inner Product in $\text{rad} \Omega_x$.

In the work [1] have been entered the concept of Subtwistor Structure on manifold of arbitrary dimension. We define an Subtwistor Structure with radical of rank $r$ at the point $x \in M$ as quadruple $(\Omega_x, D_x, \Phi_x, \beta_x)$, where $\Omega_x$ is an...
skew-symmetric 2-form with radical of dimension $r$ at the point $x$, $D_x$ is an fixed Work Subspace for $\Omega_x$, $\Phi_x$ is an Affinor associated with $\Omega_x$, and $\beta_x$ is a Radical Metric for $\Omega_x$. This Subtwistor Structure induces the Inner Product

$$(v, w)_x = \Omega_x(v, \Phi_x w) + \beta(v, w)$$

in $T_x M$.

An Subtwistor Bundle with radical of rank $r$ over manifold $M$ is an bundle $P$ over $M$ with projection $\pi : P \rightarrow M$ so that for any point $x \in M$ fibre $\pi^{-1}(x)$ is a variety of all Subtwistor Structures with radical of rank $r$ at the point $x$ as was defined above.

We describe the Subtwistor Bundle over four-dimensional sphere $S^4$ and show that $S^4$ admits only Subtwistor Bundle with radical of rank 0 or 2. A Subtwistor Bundle with radical of rank 0 over $S^4$ is a two-leaf covering of classical Subtwistor Bundle over $S^4$ and is isomorphic to $\mathbb{H}P^1 \times \mathbb{C}P^3 \times \mathbb{Z}_2$, where $\mathbb{H}P^n$ is a quaternion projective space of dimension $n$, $\mathbb{C}P^n$ is a complex projective space of dimension $n$, and $\mathbb{Z}_2$ is the discrete group $\{1, -1\}$. An Subtwistor Bundle with radical of rank 2 over $S^4$ is manifold isomorphic to $\mathbb{C}P^1 \times \mathbb{C}P^2 \times \text{SM}(2)$, where $\text{SM}(2)$ is a variety of symmetric non-degenerated positive-defined matrixes $2 \times 2$. Additionaly, the structure of Subtwistor Bundle with radical of rank 0 over $S^4$ allow to prove that $S^4$ no admits an Almost Complex Structure as well as an non-degenerated skew-symmetric 2-form.

References