SUBTWISTOR BUNDLE OVER FOUR-DIMENSIONAL SPHERE

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Let M be a Smooth manifold of arbitrary dimension $n \geq 3$ and Ω be a skew-symmetric 2-form on manifold M. A Radical of 2-form Ω at the point $x \in M$ is a subspace $rad\,\Omega_x$ of tangent space T_xM formed as

$$rad\,\Omega_x = \{v \in T_xM : \Omega_x(v, .) = 0\},$$

where $\Omega(v, .)$ denote that second vector argument can be an arbitrary vector from T_xM. Note that an radical can be also well-defined for any bilinear form at the point x. It is easy to view that at the point x always exists a tangent subspace D_x so that restriction of 2-form Ω_x to D_x be a non-degenerated 2-form, and $dim(D_x$ always be a even number for any n. This tangent subspace is called the Work Subspace.

An Affinor associated with 2-form Ω_x at the point $x \in M$ is an endomorphism Φ_x of tangent space T_xM that satisfies the following conditions:

1) $\ker \Phi_x = rad\,\Omega_x$;

2) $\Phi_x^2|_{D_x} = -id$, where id is an identity operator in D_x;

3) $\Omega_x \circ \Phi_x = \Omega_x$;

4) For all $v \in T_xM$ $\Omega_x(v, \Phi_x v) \geq 0$.

The properties of affinor Φ_x implies that bilinear form

$$\Omega_x(v, \Phi_x w), v, w \in T_xM$$

is the Inner Product in Work Subspace D_x, and equals to zero when whether v or w bellong to $rad\,\Omega_x$.

A Radical Metric for 2-form Ω_x at the point $x \in M$ is a symmetric 2-form β_x so that $rad\,\beta_x = D_x$ and restriction of β_x to $rad\,\Omega_x$ is an Inner Product in $rad\,\Omega_x$.

In the work [1] have been entered the concept of Subtwistor Structure on manifold of arbitrary dimension. We define an Subtwistor Structure with radical of rank r at the point $x \in M$ as quadruple $(\Omega_x, D_x, \Phi_x, \beta_x)$, where Ω_x is an

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skew-symmetric 2-form with radical of dimension r at the point x, D_x is an fixed Work Subspace for Ω_x, Φ_x is an Affinor associated with Ω_x, and β_x is a Radical Metric for Ω_x. This Subtwistor Structure induces the Inner Product

$$(v, w)_x = \Omega_x(v, \Phi_x w) + \beta(v, w)$$

in $T_x M$.

An Subtwistor Bundle with radical of rank r over manifold M is an bundle P over M with projection $\pi : P \mapsto M$ so that for any point $x \in M$ fibre $\pi^{-1}(x)$ is a variety of all Subtwistor Structures with radical of rank r at the point x as was defined above.

We describe the Subtwistor Bundle over four-dimensional sphere S^4 and show that S^4 admits only Subtwistor Bundle with radical of rank 0 or 2. A Subtwistor Bundle with radical of rank 0 over S^4 is a two-leaf covering of classical Subtwistor Bundle over S^4 and is isomorphic to $\mathbb{H}P^1 \times \mathbb{C}P^3 \times \mathbb{Z}_2$, where $\mathbb{H}P^n$ is a quaternion projective space of dimension n, $\mathbb{C}P^n$ is a complex projective space of dimension n, and $\mathbb{Z}_2$ is the discrete group $\{1, -1\}$. An Subtwistor Bundle with radical of rank 2 over S^4 is manifold isomorphic to $\mathbb{C}P^1 \times \mathbb{C}P^2 \times \mathrm{SM}(2)$, where $\mathrm{SM}(2)$ is a variety of symmetric non-degenerated positive-defined matrixes 2×2. Additionaly, the structure of Subtwistor Bundle with radical of rank 0 over S^4 allow to prove that S^4 no admits an Almost Complex Structure as well as an non-degenerated skew-symmetric 2-form.

References

[1] E. S. Kornev, *Subcomplex And Sub-kähler Structures* // Siberian Mathematical Journal, Vol. 57, No. 5, pp. 830-840.